3D VISION

Camera Model: Calibration

Systems Engineering and Automation

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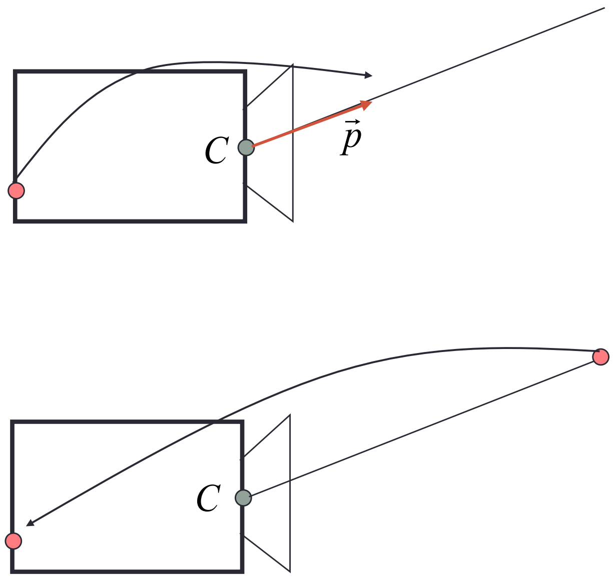
General Camera Model

- We can model the camera from the projection ray
  - It allows to include distortion or more complex projection models



Calibration

Projection

Calibration Procedure

- **CALIBRATION**: Determining the parameters involved in the image formation process:
  - Intrinsic Parameters:
    - Scale factors: K_x, K_y
    - Focal distance: f
    - Principal point: C_x, C_y
    - Distortion: D_x, D_y
  - Extrinsic Parameters:
    - Translation vector: $\boldsymbol{t}$
    - Rotation matrix: $\boldsymbol{R}$

Calibration Procedure

- **Steps in the calibration process**
  - System equations:
    - Mathematical model
  - Collecting calibration data:
    - Projection on the image of known 3D points
  - Parameters estimation:
    - Solving equations with calibration data

Projective Equations

- Matrix relationship between the world coordinates and the lateral coordinates of the camera (without distortion). **Projection Matrix**

$$\tilde{m}_f \cong \tilde{A}_1\ \tilde{A}_0 \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \tilde{M}$$

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Big\downarrow$$

$$A = \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \tilde{P} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boxed{\tilde{m} \cong [A \quad 0] \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \tilde{M}}$$

$$\boxed{\tilde{m} \cong A\ [R_c^w \quad t_c^w]\ \tilde{M}}$$

$$\boxed{\tilde{m} \cong \tilde{P}\ \tilde{M} = [P \quad p]\ \tilde{M}}$$

Projective Equations

- Components of the **Projection Matrix** (without distortion)

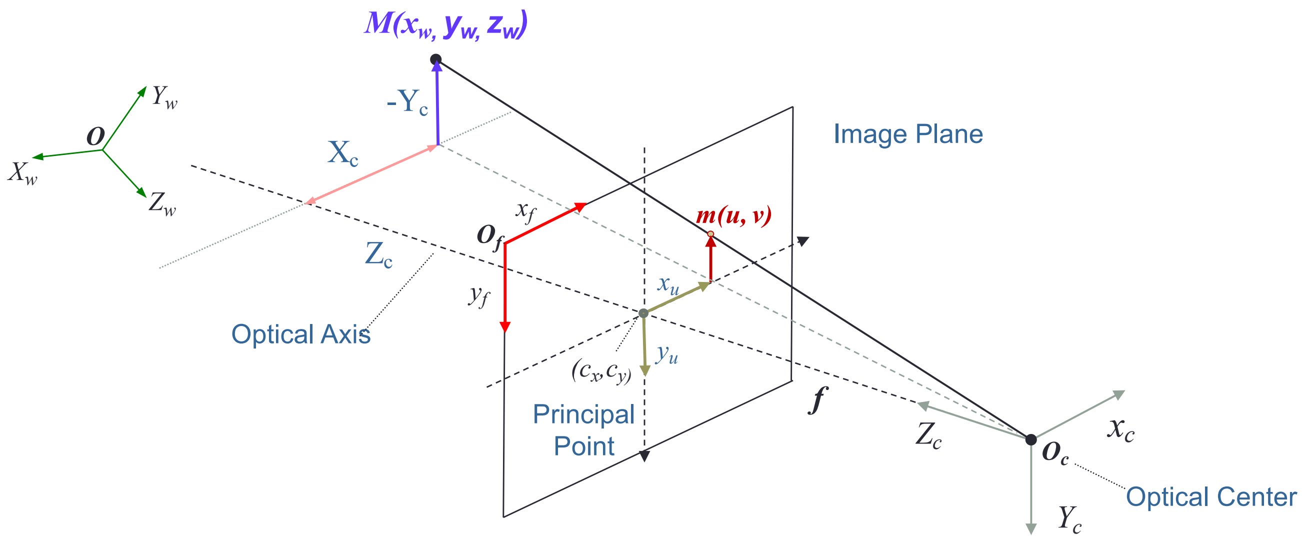
$$\begin{bmatrix} n\, x_f \\ n\, y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f\, r_{11} + C_x\, r_{31} & K_x f\, r_{12} + C_x\, r_{32} & K_x f\, r_{13} + C_x\, r_{33} & K_x f\, t_x + C_x\, t_z \\ K_y f\, r_{21} + C_y\, r_{31} & K_y f\, r_{22} + C_y\, r_{32} & K_y f\, r_{23} + C_y\, r_{33} & K_y f\, t_y + C_y\, t_z \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\text{if } \begin{cases} \bar{r}_1 = [r_{11} \quad r_{12} \quad r_{13}] \\ \bar{r}_2 = [r_{21} \quad r_{22} \quad r_{23}] \\ \bar{r}_3 = [r_{31} \quad r_{32} \quad r_{33}] \end{cases} \quad \Rightarrow \quad \begin{bmatrix} n\, x_f \\ n\, y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f\, \bar{r}_1 + C_x\, \bar{r}_3 & K_x f\, t_x + C_x\, t_z \\ K_y f\, \bar{r}_2 + C_y\, \bar{r}_3 & K_y f\, t_y + C_y\, t_z \\ \bar{r}_3 & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boxed{\tilde{m} \cong \tilde{P}\ \tilde{M} = [P \quad p]\ \tilde{M}}$$

Projective Equations

- Summary:



$$\tilde{m} \cong \tilde{P}\ \tilde{M} = [P \quad p]\ \tilde{M}$$

$$\tilde{m} \cong A\ [R \quad t]\ \tilde{M}$$

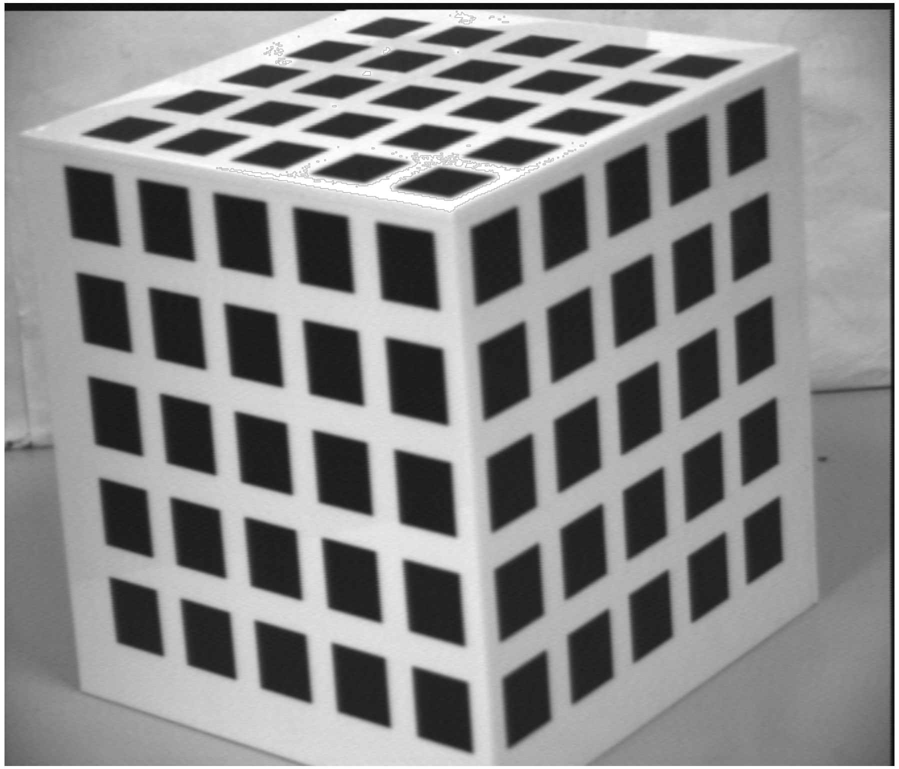
$$\tilde{m} \cong AR\ [I \quad R^T t]\ \tilde{M} = AR\ [I \quad -C]\ \tilde{M}$$

Optical Center Position:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} C \\ 1 \end{bmatrix} \Rightarrow \tilde{C} \cong \begin{bmatrix} -R^T t \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

Calibration Procedure



Calibration pattern

Calibration

- Calibration Accuracy
  - Projection parameters are estimated

$$\left(x_f, y_f\right) \approx F(parameters, X_w, Y_w, Z_w)$$

  - The true parameters are usually unknown, so they cannot be compared to the estimated parameters
  - Reprojection Error:
    - With the Estimated parameters, the points *(i)* are projected and compared to those detected in the image: $\left(x_f^{(i)}, y_f^{(i)}\right)$ versus $\left(\hat{x}_f^{(i)}, \hat{y}_f^{(i)}\right)$

$$e = \sum_i \left(\hat{x}_f^{(i)} - x_f^{(i)}\right)^2 + \left(\hat{y}_f^{(i)} - y_f^{(i)}\right)^2$$

Calibration Techniques

- Calibration Methods:
  - Techniques applied to Photogrammetry
    - Very accurate calibration methods are required
    - Professional optics without distortion problems are used
    - A priori knowledge of intrinsic parameters
  - Techniques applied to Robotics and Automation
    - Fast and autonomous methods are needed
    - Lower-resolution images
    - They are affected by many factors that systematically cause errors

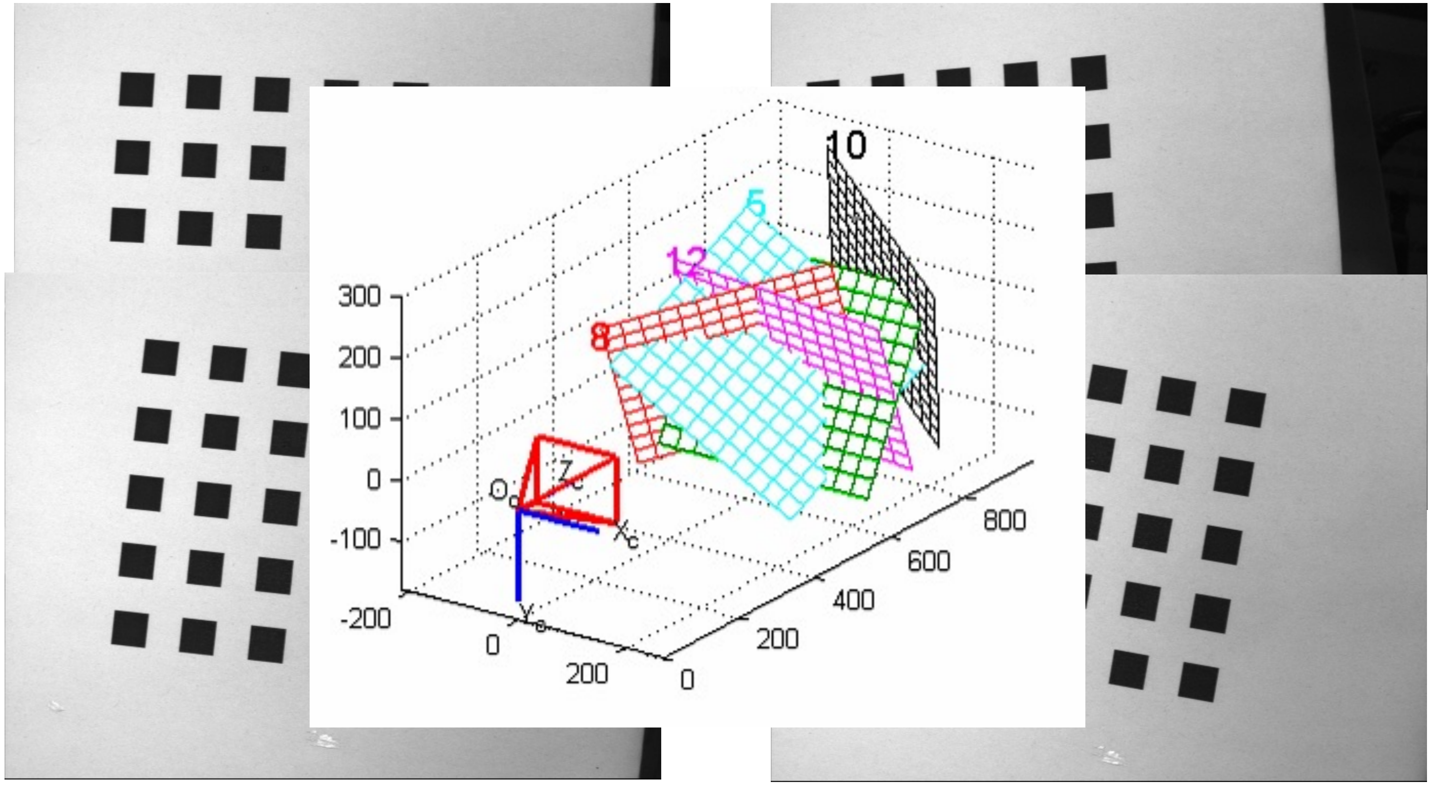
Calibration Techniques

- Techniques based on Mathematical Models
  - Mathematical relationship between 3D points and the projection of those points in the 2D image
  - The physical parameters of the camera are not take into account
  - Nonlinear optimization
- Model-based techniques with physical significance
  - Direct Linear Transformation (DLT)
    - Linear method. Distortion is calibrated a posteriori (non-linear). Requires points in a general disposition.
  - Radial Alignment Restriction (RAC) (Tsai)
    - Only radial distortion is taken into account
  - Zhang's method (OpenCV, CalibToolbox)
    - Uses multiple views of the same planar calibration pattern
    - Easier to build the calibration element and easier to cover the entire workspace

"Flexible Camera Calibration By Viewing a Plane From Unknown Orientations" Zhengyou Zhang
International Conference on Computer Vision (ICCV'99) (1999)

Calibration Techniques: Zhang's method

- Zhang (1999) proposes to calibrate with several views of the same planar pattern



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Calibration Techniques: Zhang's method

- Zhang (1999) proposes to calibrate with several views of the same planar pattern

- Advantages
  - Forces the decoupling of intrinsic (constant) from extrinsic parameters
  - Simplify the calibration element
  - Incorporate more control points distributed in more planes

- Disadvantage
  - Increases processing time (not very important when performing offline)

"Flexible Camera Calibration By Viewing a Plane From Unknown Orientations" Zhengyou Zhang
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Calibration Techniques: Zhang's method

- Unknowns
  - Four intrinsic parameters
  - Four distortion parameters
  - Six parameters per view (three from rotation and three from translation)
- Equations
  - Two for every point seen in each image

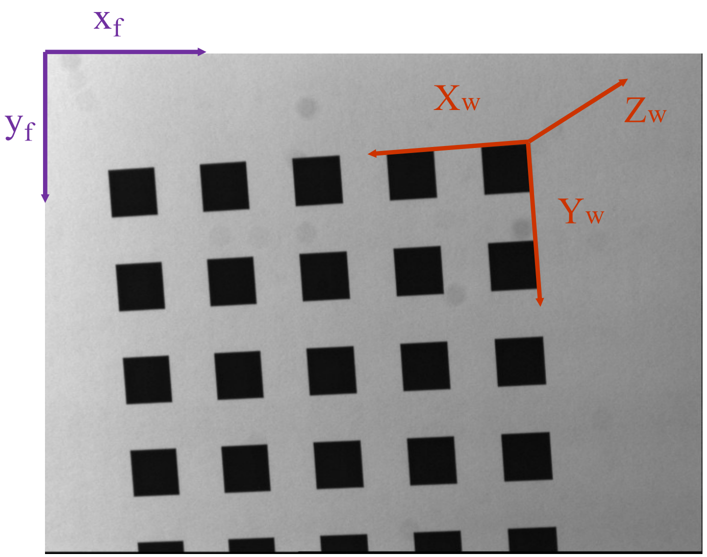
- Example: 10 images with 100 points each
  - Unknowns: 4 + 4 + 6*10 = 68 unknowns
  - Equations: 2 * 10 * 100 = 2000 equations

Calibration Techniques: Zhang's method

- Steps:
  - Calculation of the homography between image and object
  - Calculation of intrinsic parameters with multiple views
  - Calculation of extrinsic parameters for each view
  - Global optimization considering distortion

Calibration Techniques: Zhang's method

- Calculation of the Homography between image and object
  - Image coordinates $[x_f \quad y_f \quad 1]^T$
  - Coordinates of a point referring to the world's coordinate system
    $[X_w \quad Y_w \quad Z_w \quad 1]^T$
  - If the world coordinate system is chosen solidary to the calibration plane such that the third component is zero $[X_w \quad Y_w \quad 0 \quad 1]^T$



Distortion-free model

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n \cdot x_f \\ n \cdot y_f \\ n \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

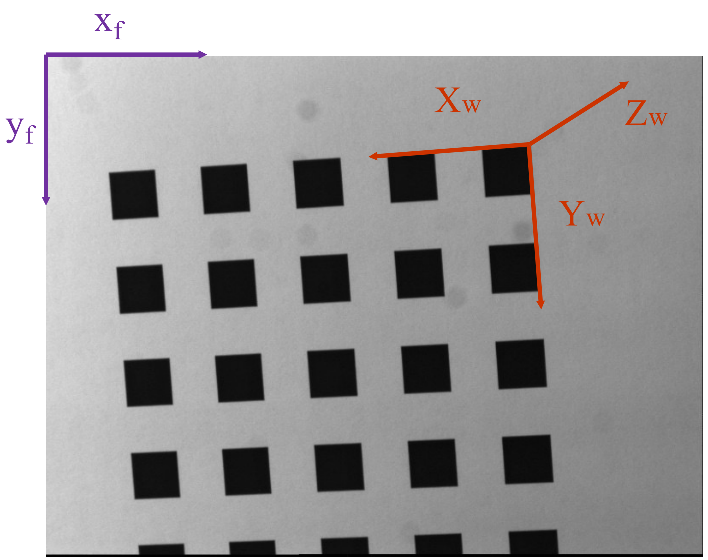
$$\boxed{H = \lambda A\,[\bar{\mathbf{r}}_1 \quad \bar{\mathbf{r}}_2 \quad \bar{\mathbf{t}}]}$$

Calibration Techniques: Zhang's method

- Calculation of the Homography :
  - The homography can be obtained with the knowledge of at least four correspondences.
  - The more correspondences, the more accurate the homography calculation is
  - Robust methods can be employed (RANSAC) in order to remove false correspondences

$$\begin{bmatrix} n \cdot x_f \\ n \cdot y_f \\ n \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

$$(h_{31}X_w + h_{32}Y_w + h_{33}) \cdot x_f = h_{11}X_w + h_{12}Y_w + h_{13}$$
$$(h_{31}X_w + h_{32}Y_w + h_{33}) \cdot y_f = h_{21}X_w + h_{22}Y_w + h_{23}$$



Homogeneous Equation Solution:

$$\mathbf{A}_{xy} = \begin{bmatrix} -X_w^{(1)} & -Y_w^{(1)} & -1 & 0 & 0 & 0 & X_w^{(1)}x_f^{(1)} & Y_w^{(1)}x_f^{(1)} & x_f^{(1)} \\ 0 & 0 & 0 & -X_w^{(1)} & -Y_w^{(1)} & -1 & X_w^{(1)}y_f^{(1)} & Y_w^{(1)}y_f^{(1)} & y_f^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\mathbf{h} = [h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}, h_{33}]^T$$

$$\mathbf{A}_{xy} \cdot \mathbf{h} = 0 \qquad \min_{\mathbf{h}} \left[(\mathbf{A}_{xy}\mathbf{h})^2 \right]$$

$$\boxed{SVD(A_{xy}) \rightarrow A_{xy} = UDV^T \Rightarrow \mathbf{h} = v_9}$$

Calibration Techniques: Zhang's method

- Calculation of the Homography:
  - Non-homogeneous solution (Euclidean): we assume $(h_{33} \neq 0)$ $H = \lambda A[\bar{\mathbf{r}}_1 \quad \bar{\mathbf{r}}_2 \quad \bar{\mathbf{t}}]$

$$\begin{bmatrix} n \cdot x_f \\ n \cdot y_f \\ n \end{bmatrix} = \begin{bmatrix} h_{11}/h_{33} & h_{12}/h_{33} & h_{13}/h_{33} \\ h_{21}/h_{33} & h_{22}/h_{33} & h_{23}/h_{33} \\ h_{31}/h_{33} & h_{32}/h_{33} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ X_w \\ 1 \end{bmatrix} \qquad h_{33} \cong t_z$$

Distance between planes

$$\begin{bmatrix} n \cdot x_f \\ n \cdot y_f \\ n \end{bmatrix} = \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ X_w \\ 1 \end{bmatrix} \Longrightarrow \begin{array}{l} (h'_{31}X_w + h'_{32}Y_w + 1) \cdot x_f = h'_{11}X_w + h'_{12}Y_w + h'_{13} \\ (h'_{31}X_w + h'_{32}Y_w + 1) \cdot y_f = h'_{21}X_w + h'_{22}Y_w + h'_{23} \end{array}$$

$$\mathbf{A}_{xy} = \begin{bmatrix} -X_w^{(1)} & -Y_w^{(1)} & -1 & 0 & 0 & 0 & X_w^{(1)}x_f^{(1)} & Y_w^{(1)}x_f^{(1)} \\ 0 & 0 & 0 & -X_w^{(1)} & -Y_w^{(1)} & -1 & X_w^{(1)}y_f^{(1)} & Y_w^{(1)}y_f^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} -x_f^{(1)} \\ -y_f^{(1)} \\ \vdots \end{bmatrix}^T$$

$$\mathbf{A}_{xy} \cdot \mathbf{h} = \mathbf{b}$$

$$\min_{\mathbf{h}} \left[\left(\mathbf{A}_{xy}\mathbf{h} - \mathbf{b} \right)^2 \right]$$

$$\mathbf{h} = [h'_{11}, h'_{12}, h'_{13}, h'_{21}, h'_{22}, h'_{23}, h'_{31}, h'_{32}]^T$$

  - Least squares: $\boxed{\mathbf{h} = \left(A_{xy}^T A_{xy}\right)^{-1} \cdot A_{xy}^T \cdot \mathbf{b}}$

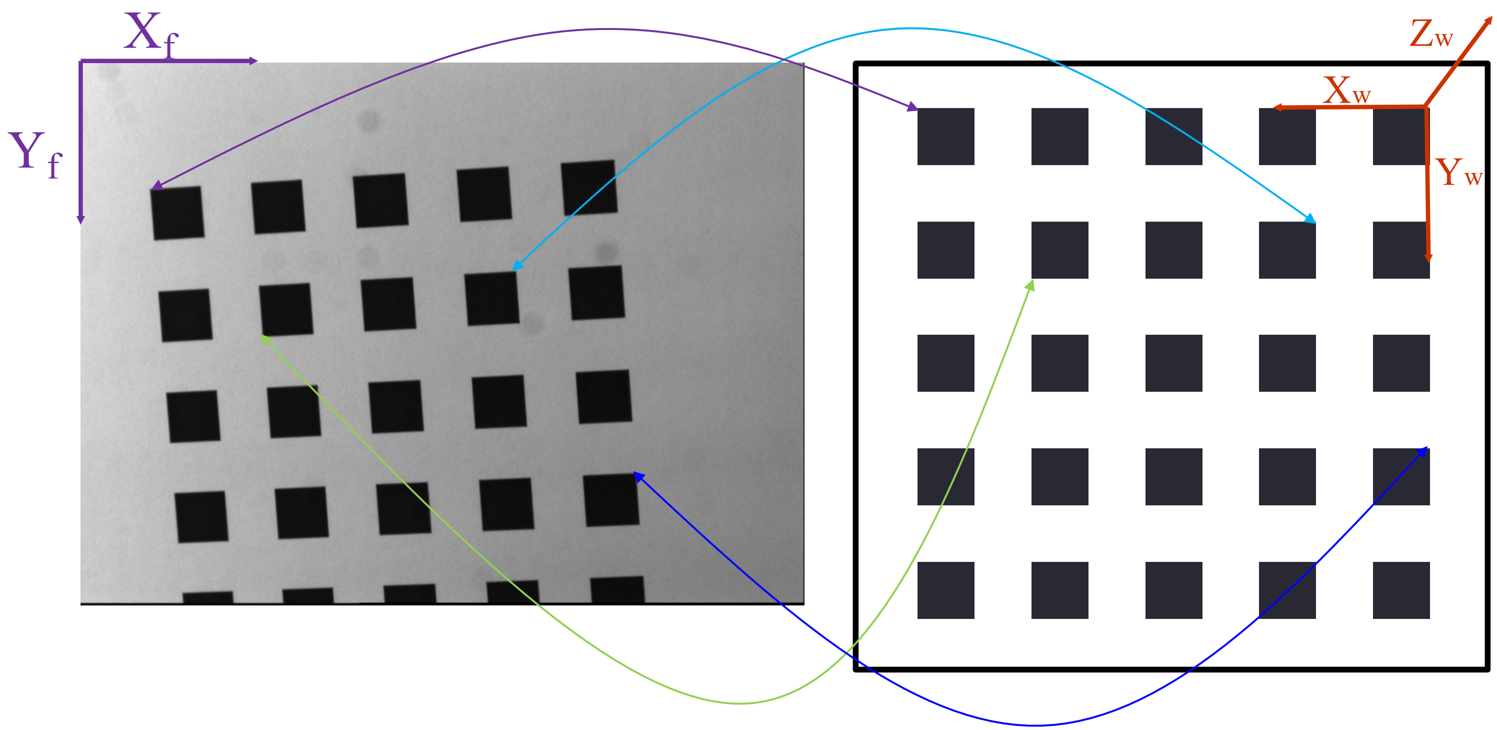
- Normalization: more robust estimation against noise

$$\left.\begin{array}{l} \tilde{\hat{m}}_w = T \cdot \tilde{m}_w \\ \tilde{\hat{m}} = T' \cdot \tilde{m} \end{array}\right] \longrightarrow \hat{m} \cong \hat{H} \cdot \hat{m}_w \longrightarrow \boxed{H \cong T'^{-1} \cdot \hat{H} \cdot T}$$

$$T^{-1} = \begin{bmatrix} n_f/2 & 0 & n_f/2 \\ 0 & n_c/2 & n_c/2 \\ 0 & 0 & 1 \end{bmatrix}$$

Calibration Techniques: Zhang's method

- Calculation of the Homography:



Calibration Techniques: Zhang's method

- Calculation of intrinsic parameters with multiple views

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \lambda \begin{bmatrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \Rightarrow$$

$$[\mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3] = \lambda A\,[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] \quad \Rightarrow \quad [\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] = \lambda^{-1} A^{-1} [\mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3]$$

$$\text{Se cumple:} \begin{cases} \mathbf{r}_1^T \mathbf{r}_2 = 0 \quad \Rightarrow \quad \mathbf{h}_1^T A^{-T} A^{-1} \mathbf{h}_2 = \mathbf{h}_1^T K^{-1} \mathbf{h}_2 = 0 \\ \mathbf{r}_1^T \mathbf{r}_1 = \mathbf{r}_2^T \mathbf{r}_2 \quad \Rightarrow \quad \begin{cases} \mathbf{h}_1^T A^{-T} A^{-1} \mathbf{h}_1 = \mathbf{h}_2^T A^{-T} A^{-1} \mathbf{h}_2 \\ \mathbf{h}_1^T K^{-1} \mathbf{h}_1 = \mathbf{h}_2^T K^{-1} \mathbf{h}_2 \end{cases} \\ \text{Con } K = A \cdot A^T \end{cases}$$

$$\text{Known } H \rightarrow \text{Two linear K constraints per view}$$

$$\boxed{\begin{cases} \mathbf{h}_1^T K^{-1} \mathbf{h}_2 = 0 \\ \mathbf{h}_1^T K^{-1} \mathbf{h}_1 = \mathbf{h}_2^T K^{-1} \mathbf{h}_2 \end{cases}}$$

Calibration Techniques: Zhang's method

- Calculation of intrinsic parameters with multiple views
  - Kruppa Matrix (Symmetrical): K (6 independent elements)
  - Absolute Conic Image: K^{-1}
  - With three views we can determine the matrix K^{-1}

$$K = AA^T = \begin{bmatrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_x f & 0 & 0 \\ 0 & K_y f & 0 \\ C_x & C_y & 1 \end{bmatrix} = \begin{bmatrix} K_x^2 f^2 + C_x^2 & C_x C_y & C_x \\ C_x C_y & K_y^2 f^2 + C_y^2 & C_y \\ C_x & C_y & 1 \end{bmatrix}$$

  - K, K^{-1} are symmetrical. Cholesky decomposition can be used to obtain intrinsic parameters

$$\boxed{Cholesky(K) = A \cdot A^T}$$

$$\boxed{\begin{array}{l} f_x = K_x f \\ f_y = K_y f \\ C_x, C_y \end{array}}$$

Calibration Techniques: Zhang's method

- Steps
  - Calculation of the homography between image and object
  - Calculation of intrinsic parameters with multiple views
  - Calculation of extrinsic parameters for each view
  - Global optimization considering distortion
- Calculation of extrinsic parameters for each view
  - Known *A, H we can calculated*:

$$[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}] = \lambda^{-1}A^{-1}[\mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3]$$

  - We get the scale factor and the translation vector
  - With only two columns you get the rotation matrix (redundant information exists)

$$\|\mathbf{r}_1\| = 1 \Rightarrow \quad \lambda^{-1} = 1/\|A^{-1}\mathbf{h}_1\|$$

$$\mathbf{t}_1 = \lambda^{-1}A^{-1}\mathbf{h}_3$$

$$\mathbf{r}_1 = \lambda^{-1}A^{-1}\mathbf{h}_1$$
$$\mathbf{r}_2 = \lambda^{-1}A^{-1}\mathbf{h}_2$$
$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Calibration Techniques: Zhang's method

- Steps
  - Calculation of the homography between image and object
  - Calculation of intrinsic parameters with multiple views
  - Calculation of extrinsic parameters for each view
  - Global optimization considering distortion
- Global optimization considering distortion
  - The previous solution can be considered definitive, and only distortion parameters are calculated
  - The previous solution is considered the initial solution of the optimization process (all parameters vary)

$$\left.\begin{aligned} x_f &= C_x + fK_x x_n + fK_x[k_1 r_n^2 x_n + k_2 r_n^4 x_n + k_3 r_n^6 x_n + 2p_1 x_n y_n + p_2(2x_n^2 + r_n^2)] \\ y_f &= C_y + fK_y y_n + fK_y[k_1 r_n^2 y_n + k_2 r_n^4 y_n + k_3 r_n^6 y_n + 2p_2 x_n y_n + p_1(2y_n^2 + r_n^2)] \end{aligned}\right\}$$

$$r_n^2 = x_n^2 + y_n^2 \quad ; \quad x_n = x_c/z_c \quad ; \quad y_n = y_c/z_c$$

$$\min_p \left\{ \sum_i \left(\hat{x}_f^{(i)} - x_f^{(i)}\right)^2 + \left(\hat{y}_f^{(i)} - y_f^{(i)}\right)^2 \right\}$$

Calibration Procedure: Conclusions

- *The calibration process can produce less robust parameters, with a high degree of instability*

- Possible causes:
  - Incomplete mathematical model
  - Errors in data acquisition
  - Poor performance of calibration algorithms
- Consequence:
  - Poor accuracy in three-dimensional vision applications