3D VISION

Camera Model

Systems Engineering and Automation

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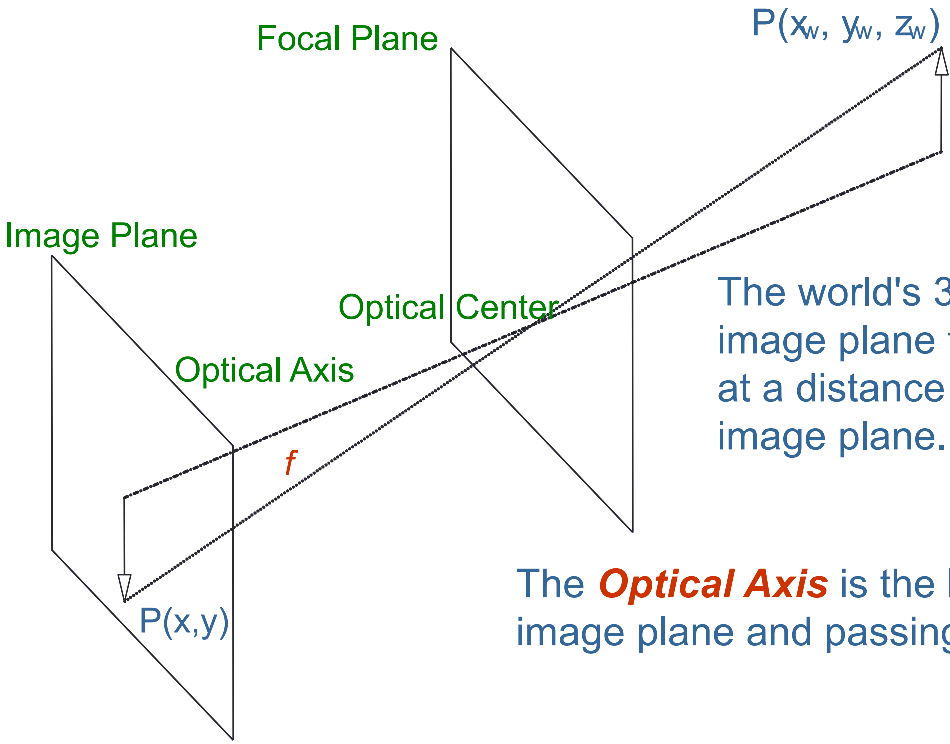
Projective Camera Model

- Pin-Hole Lens Model



The world's 3D information is projected onto the image plane through the ***Optical Center*** located at a distance equal to the ***Focal Distance*** of the image plane.

The ***Optical Axis*** is the line perpendicular to the image plane and passing through the optical center.

The ***Focal Plane*** is the plane that passes through the optical center and whose points have no projection on the plane of the image.

Geometría Proyectiva

- No conserva la distancias
- No conserva las rectas paralelas
- No conserva los ángulos

"An Introduction to Projective Geometry (for computer vision)" Stan Bircheld Stanford University http://robotics.stanford.edu/~birch/projective/

"Projective Geometry for Image Analysis" RogerMohr and Bill Triggs INRIA (ISPRS, Vienna, July 1996) https://hal.inria.fr/inria-00548361/PDF/Mohr-isprs96.pdf

Homogeneous Coordinates (Projective)

- Euclidean Space R^n: $x = [x_1, \cdots, x_n]^T \in R^n$
- Projective Space P^n $\tilde{x} = [x_1, \cdots, x_n, 1]^T \in P^n$
- In homogeneous coordinates a point is defined up to a scale factor:

$$\tilde{x} \cong \tilde{y} \quad \text{if exists} \quad \lambda \neq 0 \quad \text{so that} \quad x_i = \lambda y_i$$

$$\tilde{x} \in P^n \qquad \text{si} \quad \tilde{x} = [x_1, \cdots, x_{n+1}]^T \quad \text{with some} \quad x_i \neq 0$$

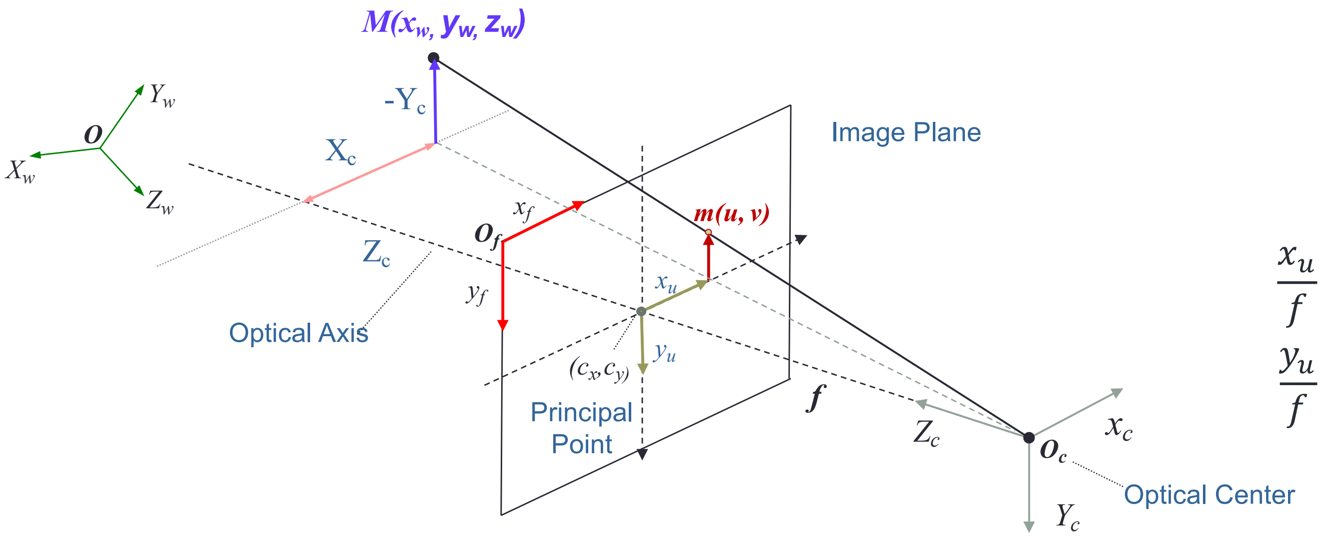
- Affine Subspace :

$$\tilde{x} = [x_1, \cdots, x_n, x_{n+1}]^T \Rightarrow x = \left[\frac{x_1}{x_{n+1}}, \cdots, \frac{x_n}{x_{n+1}}\right]^T \qquad x_{n+1} \neq 0$$

$$\tilde{x} = [x_1, \cdots, x_n, 0]^T$$ Point at infinity

$$\tilde{x} = [0, \cdots, 0]^T$$ Does not exist

Pin-Hole Lens Model



$$\frac{x_u}{f} = \frac{X_c}{Z_c} \qquad \frac{y_u}{f} = \frac{Y_c}{Z_c}$$

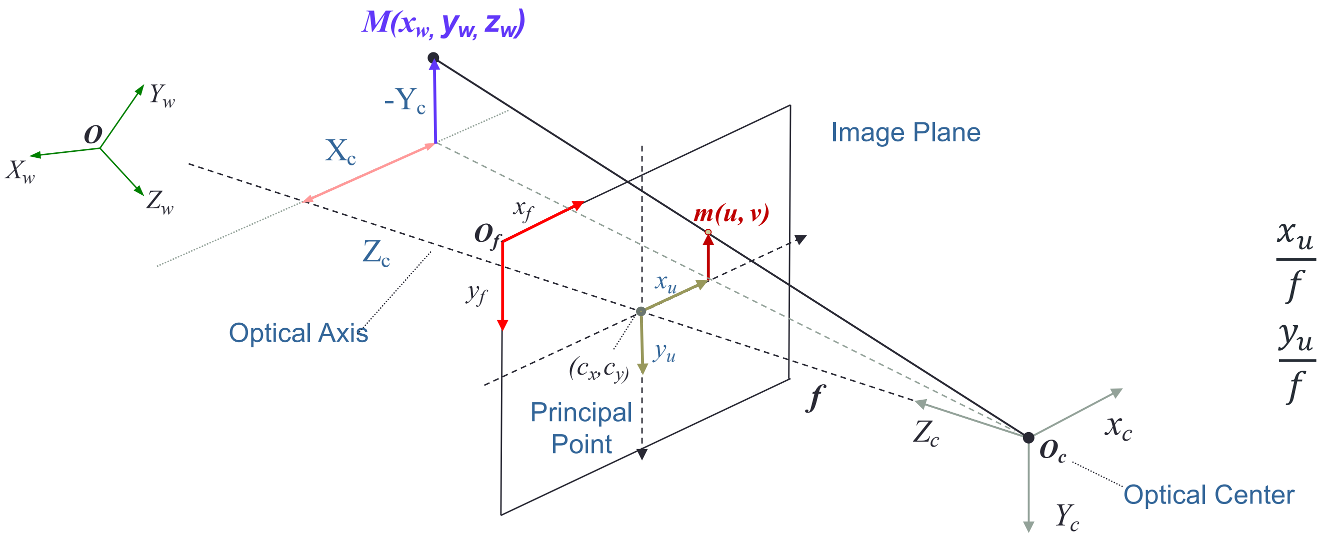
$$\begin{bmatrix} \lambda\, x_u \\ \lambda\, y_u \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & {}^{1}/_{f} & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n\, x_u \\ n\, y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$\boxed{\tilde{m}_u \cong \tilde{A}_0\ \tilde{M}_c}$$

$$\tilde{A}_0 \cong [A_0 \quad 0] \qquad A_0 \cong \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Pin-Hole Lens Model



$$\frac{x_u}{f} = \frac{X_c}{Z_c}$$

$$\frac{y_u}{f} = \frac{Y_c}{Z_c}$$

$$\begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = A_0 [I \quad 0] \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

Coordenadas 2D normalizadas: f=1

$$\begin{bmatrix} \lambda\,x_n \\ \lambda\,y_n \\ \lambda \end{bmatrix} = [I \quad 0] \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

$$\boxed{\tilde{m}_u \cong \tilde{A}_0\ \tilde{M}_c}$$

$$A_0 \cong \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

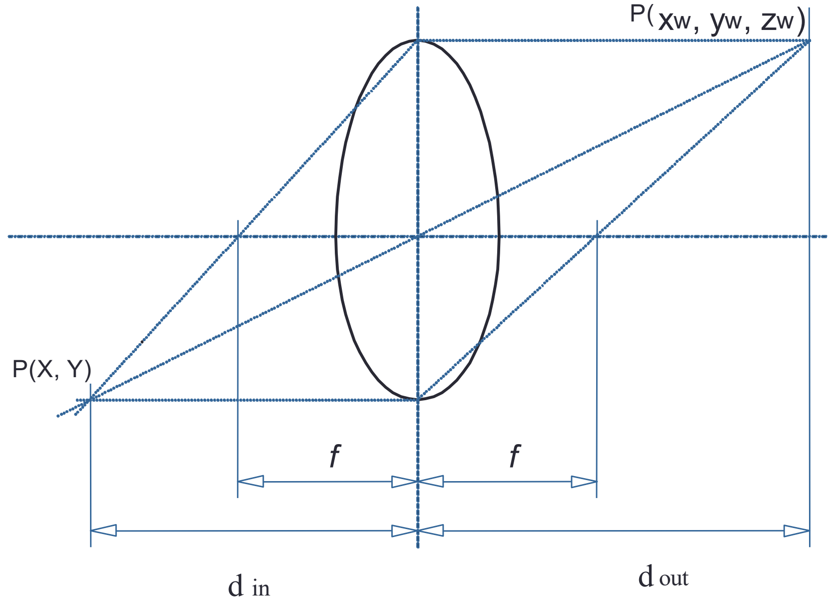
$$\boxed{\begin{array}{c} \tilde{m} \cong A_0\ \tilde{q} \\ \tilde{q} \cong A_0^{-1}\ \tilde{m} \end{array}}$$

$$x_n = \frac{X_c}{Z_c}$$

$$y_n = \frac{Y_c}{Z_c}$$

Ideal/Thin Lens Model

- Optical rays passing through the center of the lens do not suffer any deflection
- Rays that enter parallel to the optical axis of the lens converge at a point on the optical axis at a distance from the center of the lens equal to the focal length
- Produces the same projection equations as the Pin-hole model



Gauss's Law

$$\frac{1}{d_{in}} + \frac{1}{d_{out}} = \frac{1}{f}$$

Magnification

$$m = \frac{d_{\text{int}}}{d_{out}}$$

Other Projection Models

- *Orthographic Projection*
  - The rays fall perpendicularly to the plane of the image (as if they were coming from infinity). Produces a linear model
  - Valid when:
    - The viewing angle is small
    - The size of the object is small versus the distance

$$\begin{aligned} x_u &= X_c \\ y_u &= Y_c \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & f \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

Other Projection Models

- *Weak Perspective Projection*
  - Also called scaled orthographic projection
  - The object is supposed to be at a certain distance z_0

$$\frac{x_u}{f} = \frac{X_c}{z_0} \qquad \frac{y_u}{f} = \frac{Y_c}{z_0} \quad \Rightarrow \quad \begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & z_0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

Other Projection Models

- *<u>'Fisheye' Projection</u>: Scaramuzza's Model*
  - Wide-angle/omnidirectional lenses

$\begin{bmatrix} u' \\ v' \end{bmatrix}$ Distorted image coordinates (pixels)

$\begin{bmatrix} u \\ v \end{bmatrix}$ Corrected image coordinates (metric)

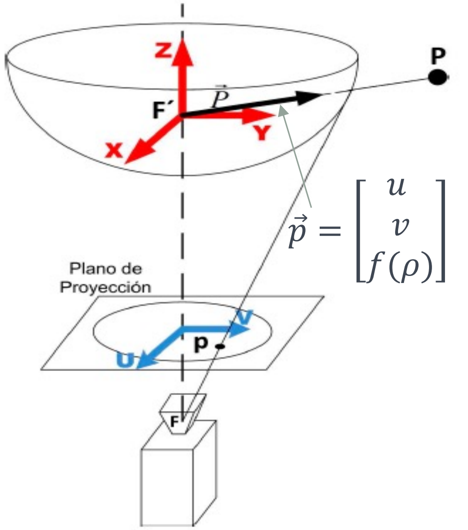
  - Model equations:

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} c & d \\ e & 1 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix}$$

$$f(\rho) = a_0 + a_1\rho + \cdots + a_n\rho^n$$
$$\rho = \sqrt{u^2 + v^2}$$

$$\vec{p} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} u \\ v \\ f(\rho) \end{bmatrix}$$

Projection beam (center F')
(Camera Coordinates)



"A Toolbox for Easy Calibrating Omnidirectional Cameras" Scaramuzza, D., A. Martinelli, and R. Siegwart
Proceedings to IEEE International Conference on Intelligent Robots and Systems, (IROS). Beijing, China, October 7–15, 2006

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- Projective Camera Model
  - Lens model: Pin-hole, Thin
  - Camera Model Parameters
  - Coordinate Systems Involved
  - Projective Equations

Camera Model Parameters

- ***Intrinsic Parameters***: Those describing the geometry and optics of the camera and the Image Acquisition Card. They affect the process that a luminous ray follows from reaching the lens until it hits a sensor element

- ***Extrinsic Parameters***: Those defining the orientation and position of the camera, relative to a known coordinate system, which will be called the *World Coordinate System (R, t)*

Intrinsic Parameters

- **Focal distance**
  - Distance separating the optical center from the plane of the image (in millimeters)

- **Scale Factors**
  - Relate the coordinates (mm) of the image plane, to the pixel coordinates of the storage memory

- **Principal Point**
  - It is the intersection point between the optical axis of the camera and the image plane

- **Distortion Coefficients**
  - Geometric distortion affects points on the image plane (optical aberrations)
  - The amount of position error depends on the position of the point on the plane of the image
- **Three types of distortion**
  - Radial Distortion
  - Tangential Distortion
  - Prismatic Distortion

Intrinsic Parameters

- *Total Distortion*
  - Distortion model in OpenCV (*Open Source Computer Vision*), http://opencv.org/

$$\left.\begin{aligned} D_x(x_n, y_n) &= \underbrace{k_1 r^2 x_n + k_2 r^4 x_n + k_3 r^6 x_n}_{\text{Radial}} + \underbrace{2p_1 x_n y_n + p_2(2x_n^2 + r^2)}_{\text{Tangential}} + \underbrace{s_1 r^2 + s_2 r^4}_{\text{Prismatic}} \\ D_y(x_n, y_n) &= k_1 r^2 y_n + k_2 r^4 y_n + k_3 r^6 y_n + 2p_2 x_n y_n + p_1(2y_n^2 + r^2) + s_1 r^2 + s_2 r^4 \end{aligned}\right\}$$

$$r^2 = x_n^2 + y_n^2$$

$$\boxed{\begin{aligned} x_d &= x_u + f \cdot D_x(x_n, y_n) \\ y_d &= y_u + f \cdot D_y(x_n, y_n) \end{aligned}}$$

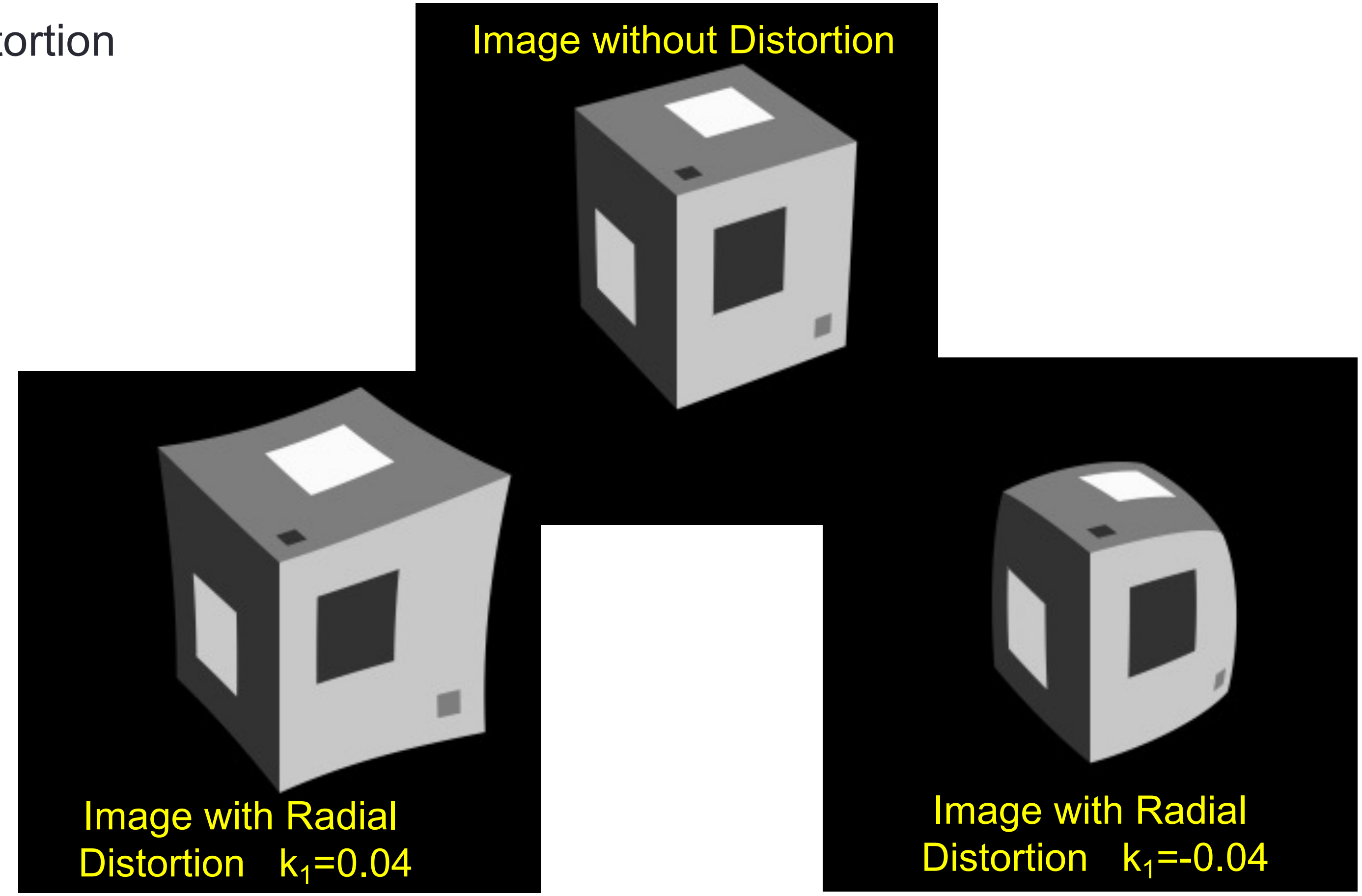
Normalized 2D coordinates: f=1

$$\left.\begin{aligned} x_n &= X_c / Z_c \\ y_n &= Y_c / Z_c \end{aligned}\right\}$$

$$\begin{bmatrix} x_u \\ y_u \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} \qquad \tilde{q} \cong A_0^{-1}\, \tilde{m}_u \cong \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix}$$

Intrinsic Parameters

- Radial Distortion



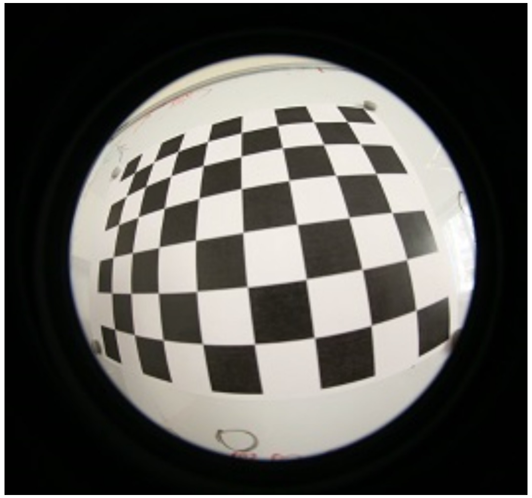
Other Distortion Models

- <u>'Fisheye' Distortion</u>: Kannala's model - OpenCV
  - Wide-angle lenses
  - Model equations:

$$\left.\begin{aligned} x_n &= X_c/Z_c \\ y_n &= Y_c/Z_c \end{aligned}\right\} \qquad \begin{aligned} r^2 &= x_n^2 + y_n^2 \\ \theta &= \operatorname{atan}(r) \end{aligned}$$

$$D_\theta(\theta) = \theta(1 + k_1\theta^2 + k_2\theta^4 + k_3\theta^6 + k_4\theta^8)$$

$$\boxed{\begin{aligned} x_d &= \frac{D_\theta(\theta)}{r} \cdot x_n \\ y_d &= \frac{D_\theta(\theta)}{r} \cdot y_n \end{aligned}}$$



"A generic camera model and calibration method for conventional, wide-angle, and fish-eye lenses" J. Kannala and S. Brandt, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 28, no. 8, pp. 1335–1340, 2006

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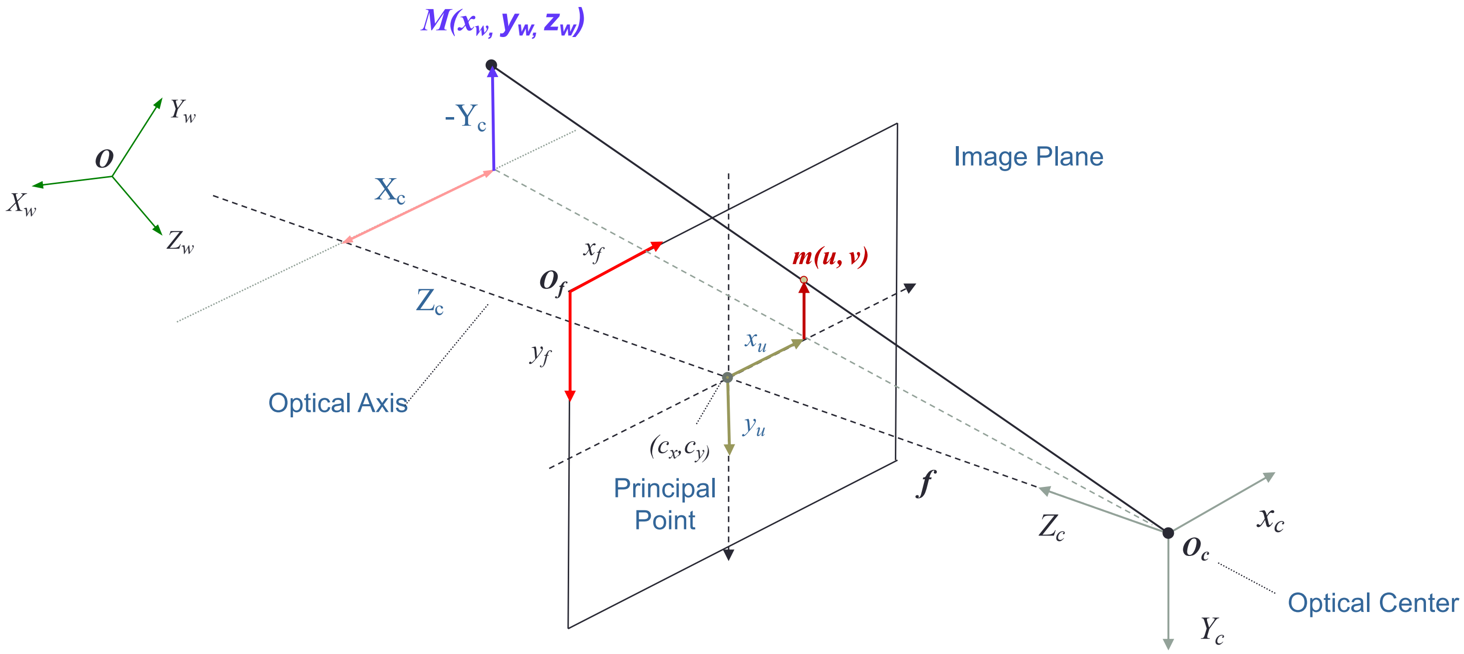
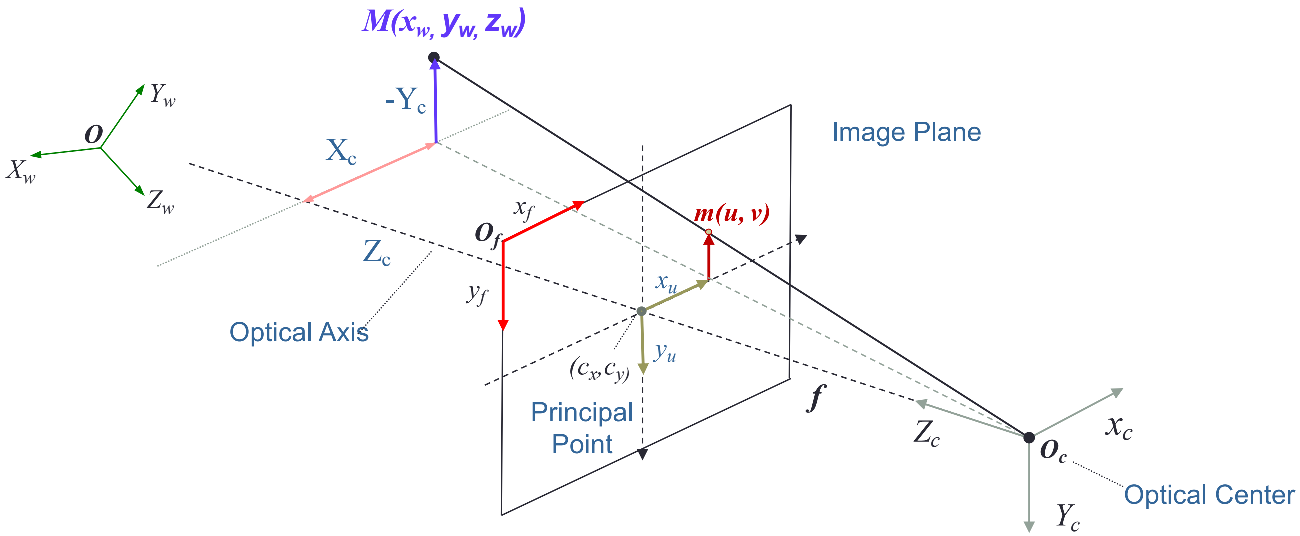
Coordinate Systems



- World Coordinates: $(O; X_w, Y_w,\ Z_w)$
- Camera Coordinates: $(O_c; X_c, Y_c,\ Z_c)$
- Central Image Coordinates: $\left(C_x, C_y; x_u, y_u\right)$
- Normalized Central Image Coordinates: $(f\ = 1)\left(C_x, C_y; x_n, y_n\right)$
- Lateral Image Coordinates: $\left(O_f; x_f, y_f\right)\ \ or\ \ \ (u, v)$

Projective Equations

- Relationship between the world coordinates and the camera coordinates:

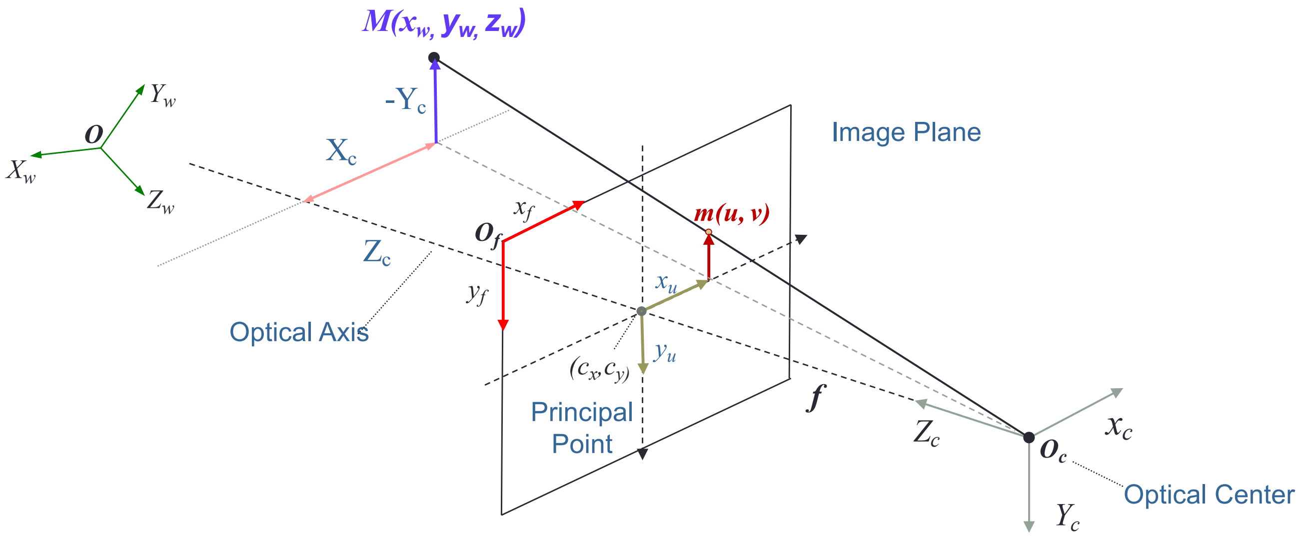


$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boxed{\widetilde{M}_C = \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \widetilde{M}_W}$$

Projective Equations

- Relationship between camera coordinates and central image coordinates:

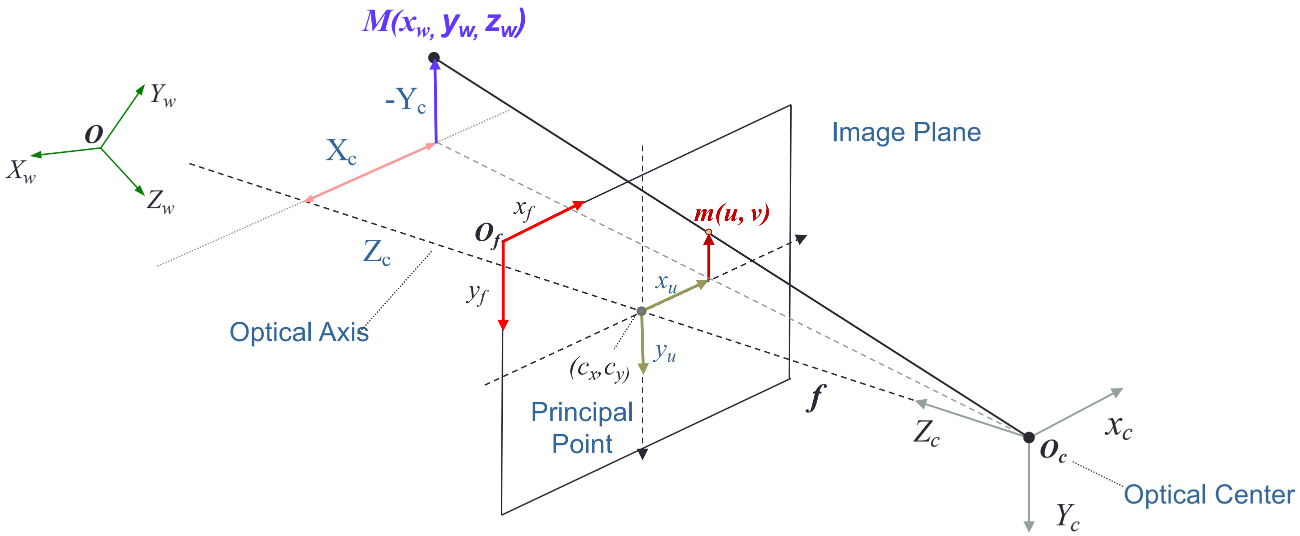


$$\begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$\boxed{\widetilde{m}_u \cong \tilde{A}_0 \; \widetilde{M}_c}$$

Projective Equations

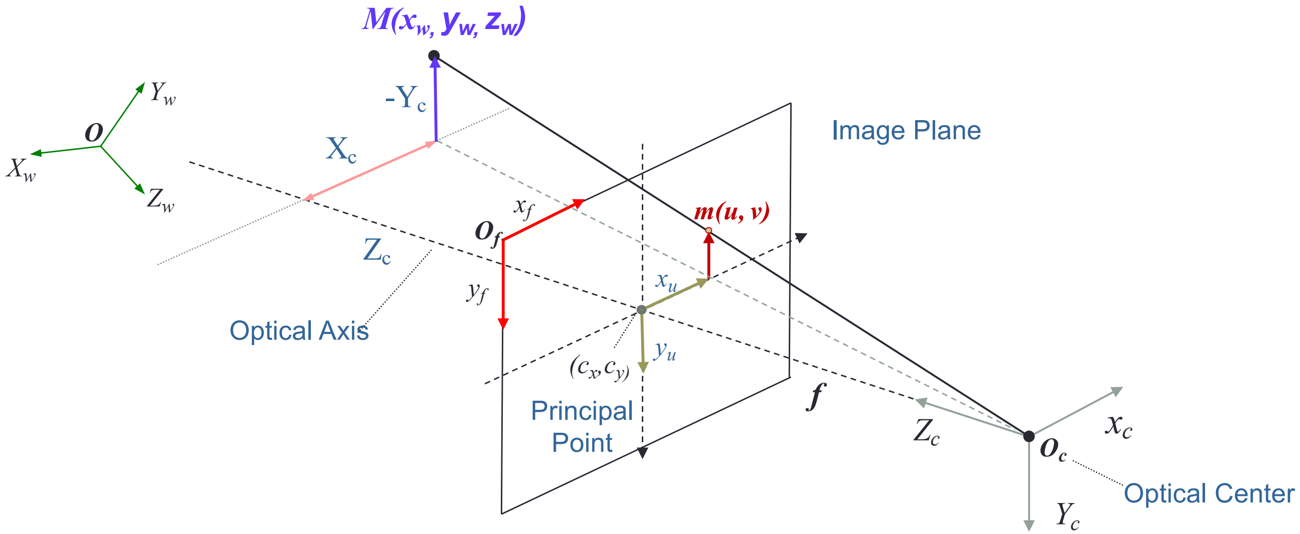
- Relationship between the *undistorted* and *distorted* central image coordinates of the image:



$$x_d = x_u + f \cdot D_x(x_n, y_n)$$
$$y_d = y_u + f \cdot D_y(x_n, y_n)$$

Projective Equations

- Relationship between the lateral and central image coordinates:

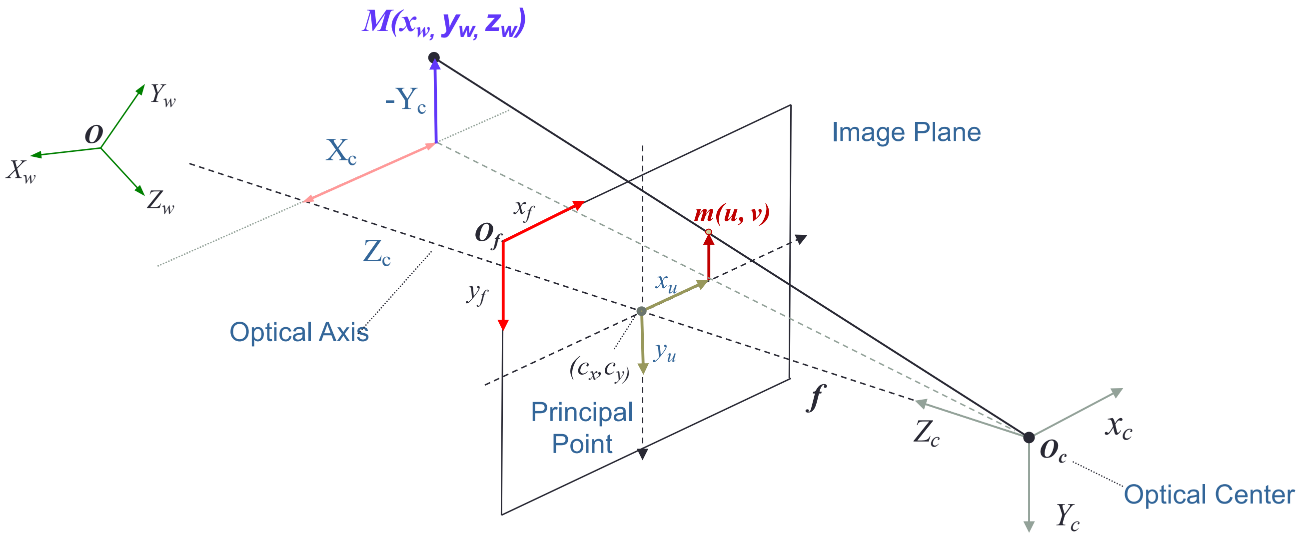


$$x_f = K_x x_d + C_x \qquad \begin{bmatrix} n\ x_f \\ n\ y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x & 0 & C_x \\ 0 & K_y & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n\ x_d \\ n\ y_d \\ n \end{bmatrix}$$
$$y_f = K_y y_d + C_y \quad \Rightarrow$$

$$\boxed{\tilde{m}_f \cong \tilde{A}_1\ \tilde{m}_d}$$

Projective Equations

- Summary of Transformations:



$$\tilde{M}_C = \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \tilde{M}_W$$

$$\tilde{m}_u \cong \tilde{A}_0 \; \tilde{M}_c$$

$$\begin{aligned} x_d &= x_u + f \cdot D_x(x_n, y_n) \\ y_d &= y_u + f \cdot D_y(x_n, y_n) \end{aligned}$$

$$\tilde{m}_f \cong \tilde{A}_1 \; \tilde{m}_d$$

Projective Equations

- Central coordinates of the camera from the world coordinates and model parameters:

$$\begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} \qquad \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} n\,x_u \\ n\,y_u \\ n \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \Rightarrow \begin{cases} x_u = f\,\dfrac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + t_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z} \\[2ex] y_u = f\,\dfrac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + t_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z} \end{cases}$$

Euclidean coordinates

Projective Equations

- Lateral coordinates of the camera from the world coordinates and model parameters:

$$
\left.\begin{aligned}
x_d &= x_u + f \cdot D_x(x_n, y_n) \\
y_d &= y_u + f \cdot D_y(x_n, y_n) \\
x_f &= K_x x_d + C_x \\
y_f &= K_y y_d + C_y
\end{aligned}\right\} \Rightarrow
\begin{cases}
x_f = C_x + K_x f \cdot D_x(x_n, y_n) + K_x x_u \\
y_f = C_y + K_y f \cdot D_y(x_n, y_n) + K_y y_u
\end{cases}
$$

$$
x_f = C_x + K_x f \cdot D_x(x_n, y_n) + K_x f \overbrace{\frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + t_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z}}^{x_n}
$$

$$
y_f = C_y + K_y f \cdot D_y(x_n, y_n) + K_y f \underbrace{\frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + t_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + t_z}}_{y_n}
$$

Projective Equations

- Matrix relationship between the world coordinates and the lateral coordinates of the camera (without distortion). **Projection Matrix**

$$\tilde{m}_f \cong \tilde{A}_1 \, \tilde{A}_0 \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \tilde{M}$$

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} K_x f & 0 & C_x \\ 0 & K_y f & C_y \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\Downarrow$$

$$A = \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \tilde{P} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\tilde{m} \cong [A \quad 0] \begin{bmatrix} R_c^w & t_c^w \\ 0 & 1 \end{bmatrix} \tilde{M}$$

$$\tilde{m} \cong A \, [R_c^w \quad t_c^w] \, \tilde{M}$$

$$\tilde{m} \cong \tilde{P} \, \tilde{M} = [P \quad p] \, \tilde{M}$$

Projective Equations

- Components of the **Projection Matrix** (without distortion)

$$\begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f\, r_{11} + C_x\, r_{31} & K_x f\, r_{12} + C_x\, r_{32} & K_x f\, r_{13} + C_x\, r_{33} & K_x f\, t_x + C_x\, t_z \\ K_y f\, r_{21} + C_y\, r_{31} & K_y f\, r_{22} + C_y\, r_{32} & K_y f\, r_{23} + C_y\, r_{33} & K_y f\, t_y + C_y\, t_z \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\text{if } \begin{cases} \bar{r}_1 = [r_{11} \quad r_{12} \quad r_{13}] \\ \bar{r}_2 = [r_{21} \quad r_{22} \quad r_{23}] \\ \bar{r}_3 = [r_{31} \quad r_{32} \quad r_{33}] \end{cases} \Rightarrow \begin{bmatrix} n\,x_f \\ n\,y_f \\ n \end{bmatrix} = \begin{bmatrix} K_x f\, \bar{r}_1 + C_x\, \bar{r}_3 & K_x f\, t_x + C_x\, t_z \\ K_y f\, \bar{r}_2 + C_y\, \bar{r}_3 & K_y f\, t_y + C_y\, t_z \\ \bar{r}_3 & t_z \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\boxed{\tilde{m} \cong \tilde{P}\ \tilde{M} = [P \quad p]\ \tilde{M}}$$

Projective Equations

- Equations considering distortion:

$$\tilde{m}_u \cong A \, [R_c^w \quad t_c^w] \, \tilde{M}_c \qquad A = \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

Normalized 2D coordinates without distortion:

$$\tilde{q} \cong A^{-1} \cdot \tilde{m}_u \cong \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix} \Longrightarrow$$

Distortion equations:

$$\left.\begin{aligned} x_n^d &= x_n(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 x_n y_n + p_2(2x_n^2 + r^2) + s_1 r^2 + s_2 r^4 \\ y_n^d &= y_n (1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_2 x_n y_n + p_1(2y_n^2 + r^2) + s_1 r^2 + s_2 r^4 \\ r^2 &= x_n^2 + y_n^2 \end{aligned}\right\}$$

Distorted-standard 2D coordinate:

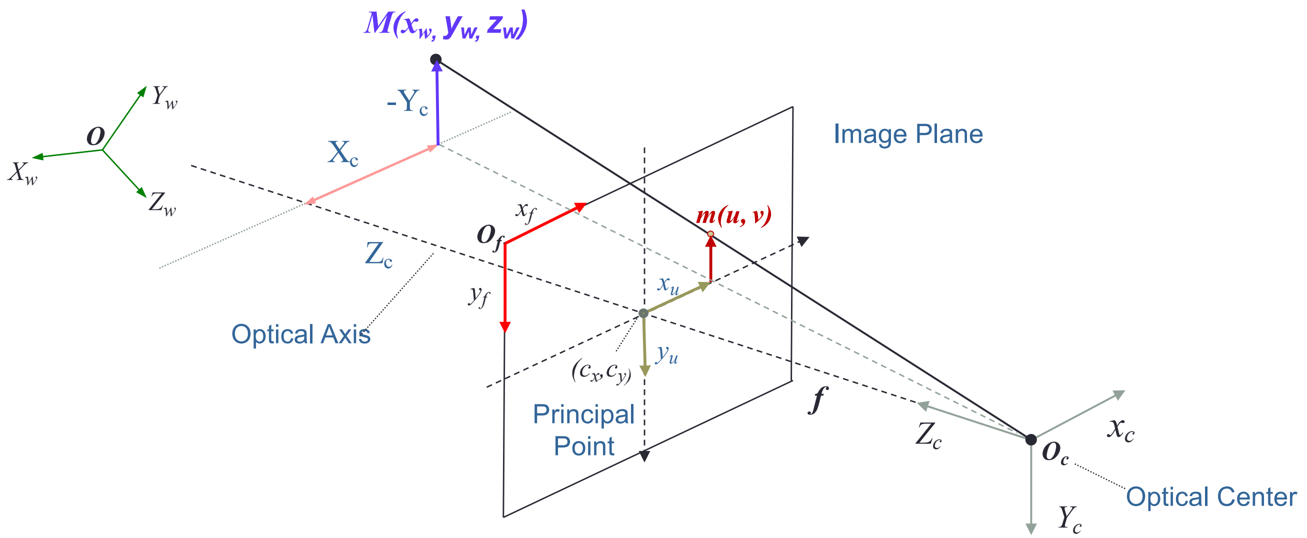
$$\tilde{q}^d \cong \begin{bmatrix} x_n^d \\ y_n^d \\ 1 \end{bmatrix}$$

Distorted 2D coordinates

$$\boxed{\tilde{m} \cong A \cdot \tilde{q}^d}$$

Projective Equations

- Summary:



$$\tilde{m} \cong \tilde{P}\ \tilde{M} = [P \quad p]\ \tilde{M}$$

$$\tilde{m} \cong A\ [R \quad t]\ \tilde{M}$$

$$\tilde{m} \cong AR\ [I \quad R^T t]\ \tilde{M} = AR\ [I \quad -C]\ \tilde{M}$$

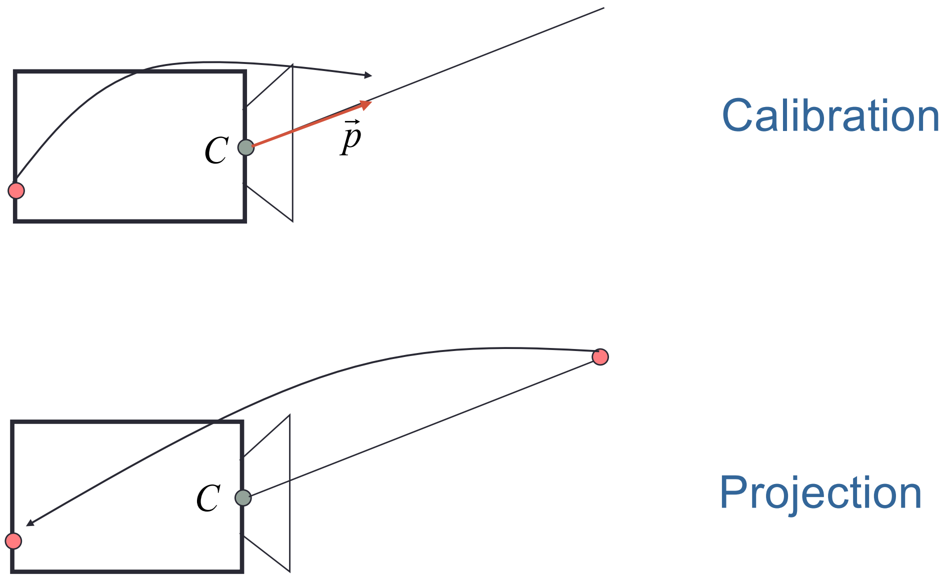
Optical Center Position:

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} C \\ 1 \end{bmatrix} \Rightarrow \tilde{C} \cong \begin{bmatrix} -R^T t \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} f_x & 0 & C_x \\ 0 & f_y & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

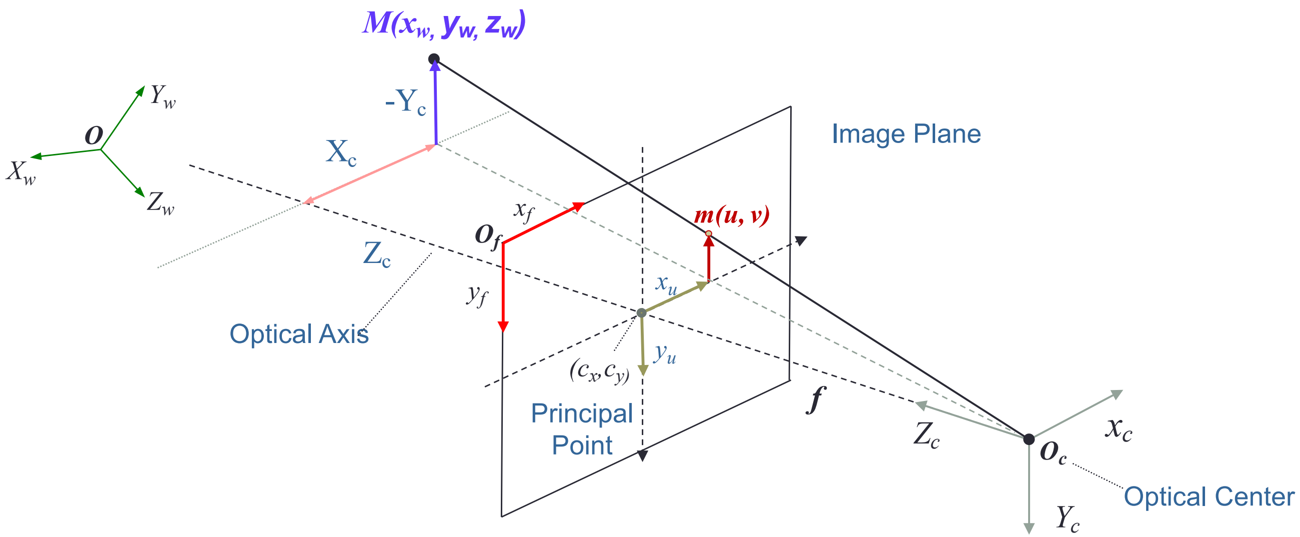
General Camera Model

- We can model the camera from the projection ray
  - Allows to include distortion or more complex projection models



Projective Equations

- Projection ray from an image point (no distortion):



Projection Ray in Camera Coordinates:

$$\tilde{m} \cong A\ [R \quad t]\ \tilde{M}$$

$$\tilde{q} \cong A^{-1} \cdot \tilde{m} \cong [R \quad t]\ \tilde{M}$$

$$\vec{x}_c = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R \quad t] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = R \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + t$$

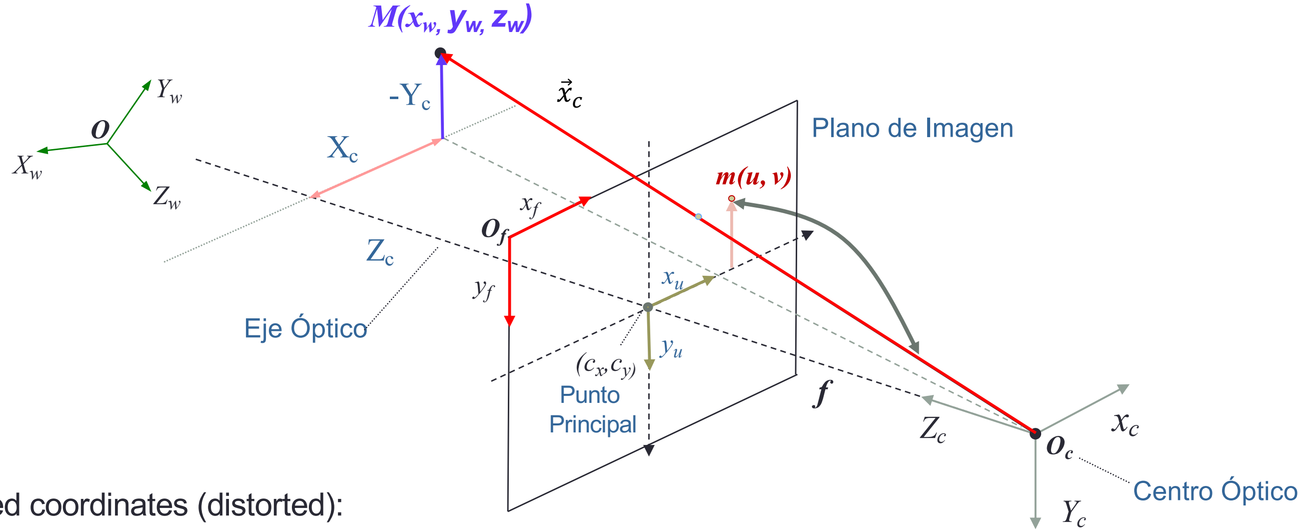
Normalized 2D projected coordinates:
origin principal point, f=1:

$$\tilde{q} = A^{-1} \cdot \tilde{m} \cong \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

$$\boxed{\vec{x}_c \cong A^{-1} \cdot \tilde{m}}$$

Projective Equations

- Ray projection from an image point (with distortion, OpenCV model):



Normalized 2D projected coordinates (distorted):

$$\tilde{q}^d \cong A^{-1} \cdot \tilde{m} \cong \begin{bmatrix} x_n^d \\ y_n^d \\ 1 \end{bmatrix}$$

$$\left.\begin{aligned} x_n^d &= x_n(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_1 x_n y_n + p_2(2x_n^2 + r^2) + s_1 r^2 + s_2 r^4 \\ y_n^d &= y_n\,(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) + 2p_2 x_n y_n + p_1(2y_n^2 + r^2) + s_1 r^2 + s_2 r^4 \\ r^2 &= x_n^2 + y_n^2 \end{aligned}\right\}$$

Distortion equations $\Rightarrow$ $\tilde{q} \cong \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix}$

Projection Ray in Camera Coordinates:

$$\vec{x}_c = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = [R \quad t]\, \tilde{M}$$

$$\boxed{\vec{x}_c \cong \tilde{q} \cong \begin{bmatrix} x_n \\ y_n \\ 1 \end{bmatrix}}$$

2D coordinates (undistorted):

$$\tilde{m}_u \cong A \cdot \tilde{q}$$

Table of Contents

- Projective Camera Model
- Next topic:
  - Calibration
  - Zhang's algorithm (OpenCV/CalibToolbox)